GCE Examinations

Advanced Subsidiary / Advanced Level

# Further Pure Mathematics
# Module FP1

Paper C

# MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



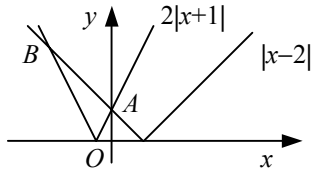

*Written by Shaun Armstrong & Chris Huffer*

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# FP1 Paper C – Marking Guide

**1.**




B2

by inspection, at $A$ $x = 0$ B1
at $B$ $-(x-2) = -2(x+1)$ giving $x = -4$ M1 A1
using graphs, require $-4 < x < 0$ A1 **(6)**

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**2.** *(a)* $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ M1 A1

$x.vx.(v + x\dfrac{dv}{dx}) = x^2 + (vx)^2 \quad \therefore xv\dfrac{dv}{dx} = 1$ M1 A1

$\int v\ dv = \int \dfrac{1}{x}\ dx$ M1

$\frac{1}{2}v^2 = \ln|x| + c$ A1

giving e.g. $y^2 = x^2(k + \ln x^2)$ A1

*(b)* $x = 1, y = 2 \quad \therefore k = 4$ M1
$x > 0 \ \therefore y^2 = x^2(4 + 2\ln x) = 2x^2(\ln x + 2)$ A1 **(9)**

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**3.** *(a)* series sum $= \sum_{r=1}^{n} (2r)^3 = 8 \times \sum_{r=1}^{n} r^3 = 8 \times \frac{1}{4}n^2(n+1)^2 = 2n^2(n+1)^2$ M2 A1

*(b)* series sum $= \sum_{r=1}^{2n} r^3 - 2 \times \sum_{r=1}^{n} (2r)^3$ M1 A1

$= \frac{1}{4}(2n)^2(2n+1)^2 - 2 \times 2n^2(n+1)^2$ M1

$= n^2[(2n+1)^2 - 4(n+1)^2]$ A1

giving $-n^2(4n+3)$ M1 A1 **(9)**

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**4.** aux. eqn. $m^2 - 6m + 9 = 0$ M1

$(m-3)^2 = 0 \quad \therefore m = 3$ A1

C.F. $y = (A + Bx)e^{3x}$ A1

P.I. try $y = Cx^2e^{3x}$ M1

$\dfrac{dy}{dx} = 3Cx^2e^{3x} + 2Cxe^{3x}, \quad \dfrac{d^2y}{dx^2} = 9Cx^2e^{3x} + 12Cxe^{3x} + 2Ce^{3x}$ M1 A2

$\therefore 9Cx^2e^{3x} + 12Cxe^{3x} + 2Ce^{3x} - 18Cx^2e^{3x} - 12Cxe^{3x} + 9Cx^2e^{3x} = 2e^{3x}$ M1

giving $2Ce^{3x} = 2e^{3x} \quad \therefore C = 1$ A1

gen. soln. $y = (A + Bx)e^{3x} + x^2e^{3x} = e^{3x}(A + Bx + x^2)$ A1 **(10)**

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**5.** *(a)* $f(1) = 0.443$; $f(1.5) = -11.1$
f cont. over interval, change of sign ∴ root — M1 A1

*(b)* $f'(x) = 2 - \sec^2 x$ — M1

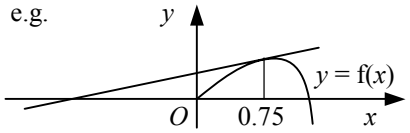
$x_{n+1} = x_n - \dfrac{2x_n - \tan x_n}{2 - \sec^2 x_n}$ — A1

$x_0 = 1.25$, $x_1 = 1.1868$, $x_2 = 1.1670$, $x_3 = 1.1656$, $x_4 = 1.1656$ — M1 A1

$\alpha = 1.17$ (2dp) — A1

$f(1.165) = 0.00248$; $f(1.175) = -0.0432$
change of sign ∴ root ∴ accurate to 2dp — M1 A1

*(c)* e.g.




B1

N-R uses intersec. of tangent at initial value with $x$-axis as next approx. — B1
at $x = 0.75$ this tangent is further away from root — B1 **(12)**

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**6.** *(a)* $z = 15 - 9i + iw$ ∴ $45 - 27i + 3iw + w = 14$ — M1
$w(1 + 3i) = -31 + 27i$ — M1
$\therefore w = \frac{-31+27i}{1+3i} \times \frac{1-3i}{1-3i} = \frac{50+120i}{10} = 5 + 12i$ — M1 A1
$3z = 14 - (5 + 12i) = 9 - 12i$ ∴ $z = 3 - 4i$ — M1 A1

*(b)* $(p + iq)^2 = 3 - 4i$; $p$, $q$ real — M1
$p^2 + 2pqi - q^2 = 3 - 4i$ — M1
$\therefore p^2 - q^2 = 3$; $2pq = -4$ — A1
e.g. sub. for $q$ giving $(p^2 - 4)(p^2 + 1) = 0$ — M1 A1
$p^2 = -1$ (no solns) or 4 ∴ $p = \pm 2$ — M1
sub. in giving $(2 - i)$ and $(-2 + i)$ — A1 **(13)**

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**7.** *(a)* $4 \sin 2\theta = 4 \cos \theta$
$4 \cos \theta (2 \sin \theta - 1) = 0$ — M1
$\sin \theta = \frac{1}{2}$ or $\cos \theta = 0$ giving $\theta = \frac{\pi}{6}$ or $\frac{\pi}{2}$ — M1 A1
at $P$, $\theta = \frac{\pi}{6}$ ∴ $P$ is $(2\sqrt{3}, \frac{\pi}{6})$ — M1 A1

*(b)* area $= \frac{1}{2}\int_0^{\frac{\pi}{6}} (4 \sin 2\theta)^2 \, d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4 \cos \theta)^2 \, d\theta$ — M1 A1

$= 4\int_0^{\frac{\pi}{6}} 1 - \cos 4\theta \, d\theta + 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1 + \cos 2\theta \, d\theta$ — M1 A2

$= 4[\theta - \frac{1}{4}\sin 4\theta]_0^{\frac{\pi}{6}} + 4[\theta + \frac{1}{2}\sin 2\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ — M1 A2

$= 4[(\frac{\pi}{6} - \frac{\sqrt{3}}{8}) - (0) + (\frac{\pi}{2}) - (\frac{\pi}{6} + \frac{\sqrt{3}}{4})]$ — M1 A1

$= 4[\frac{\pi}{2} - \frac{3\sqrt{3}}{8}] = 2\pi - \frac{3}{2}\sqrt{3}$ — A1 **(16)**

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Total **(75)**

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# Performance Record – FP1 Paper C

| Question no. | **1** | **2** | **3** | **4** | **5** | **6** | **7** | Total |
|---|---|---|---|---|---|---|---|---|
| Topic(s) | inequality | 1st order diff. eqn | series | 2nd order diff. eqn | numerical methods | complex nos. | polar coords | |
| Marks | 6 | 9 | 9 | 10 | 12 | 13 | 16 | 75 |
| Student | | | | | | | | |
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